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Heat transfer of dilute spray impinging on hot surface (simple model focusing on rebound motion and sensible heat of droplets)

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Abstract

In the present paper, focusing on the effects of the rebound motion and sensible heat of droplets on spray-cooling heat transfer in the high temperature region, a simple model was developed to predict the heat flux distribution of a dilute spray impinging on a hot surface. In the model, the local heat flux was regarded as the sum of the heat flux components by droplets, induced air flow, and radiation. To estimate the heat flux component by droplets, it was assumed that the heat flux upon droplet impact is proportional to the sensible heat which heats up the droplet to the saturation temperature and the proportional factor C is constant. In addition, to take account of the contribution of the heat flux upon impact of rebounded droplets, it was assumed that the flight distance of droplets during rebound motion is distributed uniformly from 0 to L_{max} (maximum flight distance). The values of C and L_{max} determined by experimental data of local heat flux indicate that the assumptions employed in the present model is valid at least as the first order approximation. © 1998 Elsevier Science Ltd. All rights reserved.

Nomenclature

a constant in equations (16) and (18)
b constant in equations (16) and (18)
c_p specific heat of liquid [J kg^{-1} K^{-1}]
C constant in equation (4)
d diameter of surface [m]
d_d diameter of a droplet [m]
d_m mean diameter of droplets [m]
D_{max} droplet flow rate supplied from spray to surface at $x = 0$ [m^3 m^{-2} s^{-1}]
D_x droplet flow rate supplied from spray to surface at x [m^3 m^{-2} s^{-1}]
$D_x[j]$ flow rate of droplets supplied to surface at x after (j-1)th rebound [m^3 m^{-2} s^{-1}]
h_a heat transfer coefficient by induced air flow [W m^{-2} K^{-1}]
j number of impactions of each droplet
L flight distance of droplet during rebound motion [m]
L_{max} maximum value of L [m]
$n_x[j]$ number of droplets supplied to surface after ($j-1$)th rebound [1 s^{-1}]
$q_w[x]$ local heat flux at x [W m^{-2}]
$q_{wa}[x]$ heat flux by induced air flow at x [W m^{-2}]
$q_{wl}[x]$ heat flux upon impact of droplet at x [W m^{-2}]
$q_{wr}[x]$ radiative heat flux at x [W m^{-2}]
$q_{wx}[j]$ droplet heat flux upon impact of droplet after (j-1)th rebound at x [W m^{-2}]
$Q[j]$ amount of heat transferred to each droplet upon jth impact [J]
Q_d total droplet flow rate [m^3 s^{-1}]
r radial distance from stagnation point in three dimensional system [m]
R nondimensional radius $= r/b$
T_a temperature of air flow [K]
T_{rm} room temperature [K]
T_w temperature of heat transfer surface [K]

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v velocity of droplet [m s^{-1}]
W_l total flow rate of liquid film flow [m^3 s^{-1}]
x distance from stagnation point in two-dimensional system [m]
X nondimensional distance $= x/b$.

Greek symbols
ΔT_{sat} surface superheat [K]
ΔT_{sub} subcooling of droplet [K]
ε emmisivity
ρ density of liquid [kg m^{-3}]
σ Stefan–Boltzmann constant.

1. Introduction

Among heat transfer phenomena, there are many issues which are still open because they include complicated processes. Heat transfer of a spray impinging on a hot surface is one of the typical examples of such issues. In general, spray-cooling heat transfer has the low, middle, and high temperature regions corresponding to the nucleate, transition, and film boiling regions in boiling heat transfer, respectively. Spray-cooling heat transfer in the high temperature region and the minimum heat-flux point are of great importance to attain stable and uniform cooling in the so-called Thermo-Mechanical Controlled Process (TMCP) of steel products. In particular, a good understanding of the heat flux distribution is very important to suppress the deformation of steel plates in the TMCP.

The present report is the fourth one of the present study investigating the heat flux distribution in the high temperature region of spray-cooling heat transfer with large subcooling. In the first report [1], an experimental study was conducted to measure the heat flux distribution of a two-dimensional dilute spray impinging on a flat surface. Based on the experimental results, they classified the heat transfer area into the stagnation and wall-flow regions. In the stagnation region the local heat flux relates mainly to the droplet flow rate supplied from the spray, D_x [m^3 m^{-2} s^{-1}], and in the wall-flow region it relates to the total flow rate of liquid remaining on the surface, W_l [m^3 s^{-1}]. In the second report [2], the heat flux distribution was experimentally examined in the wall-flow region where the accumulation of remaining droplets results in the formation of a liquid film flowing along the surface. In the experiments, the film flow formed by a two-dimensional spray was simulated by a forced liquid-film flow along the surface, and it was found that the local heat-transfer coefficient can be correlated by an empirical equation similar to that of turbulent heat transfer in a channel without phase change. In the third report [3], to examine the heat flux distribution in the multistage spray-cooling zone, the two-dimensional spray was supplied to the liquid film flowing along the surface, and it was found that the liquid-film flow decreased the local heat flux of spray cooling.

One of the difficulties in understanding spray-cooling heat transfer is the complicated behavior of droplets such as deformation, split, rebound, and coalescence. In addition, droplets supplied from a spray to the surface have the distributions in diameter, velocity, and number density. Complicated nature makes difficult analytical approaches for spray-cooling heat transfer. There are indeed some analytical approaches for spray-cooling heat transfer in the high temperature region. Toda [4, 5] and Bolle and Moureau [6] proposed sophisticated heat-transfer models upon impact of a droplet. Deb and Yao [7] developed a model including droplet evaporation and convective heat transfer induced by droplets, and Ito et al. [8, 9] reported a model focusing on heat transfer to droplets sliding on the surface. It is still difficult, however, to predict the local heat flux of spray-cooling heat transfer because these existing models neglect the rebound motion and flow rate distribution of droplets. In the model by Deb and Yao, the F factor was used to take account of multiple impactions of droplets but it was deduced from experimental data of the heat flux averaged over the surface.

In the present report, focusing on the high temperature region of a dilute spray with large subcooling, a simple model was developed to predict the local heat flux. In developing heat transfer models of spray cooling in the high temperature region, there are the following two possible ways. The one focuses on the evaporation process of droplets on the surface and in this type of model the latent heat of liquid plays an important role [8, 9]. The other focuses on the sensible heat which heats up the droplets to the saturation temperature [10]. In the present report, following the pioneering study by Shoji et al. [10], the latter way was chosen to develop a heat transfer model because in most of actual cases in steel processing highly subcooled water is used as the dispersed phase in spray cooling.

2. Model formulation

2.1. Conditions and assumptions

In the present paper, a heat transfer model is formulated for spray cooling of a hot flat plate facing upward under the following conditions.

(1) The droplet flow rate of the spray, D_x, is so small that each droplet can hit directly the heat transfer surface.
(2) The subcooling of droplets supplied from the spray, ΔT_{sub}, is very large.
(3) The temperature of the heat transfer surface, T_w, is uniform.

To formulate the heat transfer model, the following assumptions were employed.

(a) In general, the local heat flux, $q_w[x]$, consists of the heat flux directing to droplets, $q_{wl}[x]$, convective heat flux by induced air flow, $q_{wa}[x]$, and radiative heat flux, $q_{wr}[x]$. Here, x is the distance measured from the stagnation point of spray flow. There is interrelation among these three components. In actual cases, the droplets are heated by surface radiation before their impact and thus $q_{wr}[x]$ affects the value of $q_{wl}[x]$ through a change of the droplet temperature. The liquid mass remaining on the surface, which is determined by $q_{wl}[x]$, affects the effective heat transfer area for radiative and convective heat transfer. In the present model, we neglected such interrelation among the heat flux components.
(b) Droplets repeat the impact-rebound motion on the heat transfer surface as shown in Fig. 1. It is assumed that heat transfer to the droplets occurs only upon impact and the amount of heat transferred to each droplet upon the jth impact, $Q[j]$, is proportional to the sensible heat which heats up the droplet to the saturation temperature. It is also assumed that the proportional factor, C, is constant.
(c) The flight distance is defined as the distance between the points upon the first and second impacts of each droplet. It is assumed that the flight distance is uniformly distributed from 0 to L_{max}. L_{max} is the maximum flight distance and it is independent of the velocity and diameter of the droplet.

Assumption (a) seems reasonable for dilute sprays mentioned in condition (1). Assumption (b) was employed based on very useful results reported by Shoji et al. [10]. They conducted heat transfer experiments from a hot surface to a single droplet with large subcooling. Their experiments covered the following regions: for water droplets, $d_d = 0.88$–3.53 mm, $v = 0.89$–1.90 m s^{-1} and $\Delta T_{sub} = 41$–98 K, and for ethanol, $d_d = 1.51$–2.54 mm, $v = 1.23$–1.80 m s^{-1} and $\Delta T_{sub} = 45$–52 K. They found that Q [1] is proportional to the sensible heat which heats up the droplet to the saturation temperature and the value of C is almost independent of the velocity, diameter, and subcooling of droplets. Assumption (c) is the fundamental idea in the present model, and its validity will be checked by calculated results mentioned later.

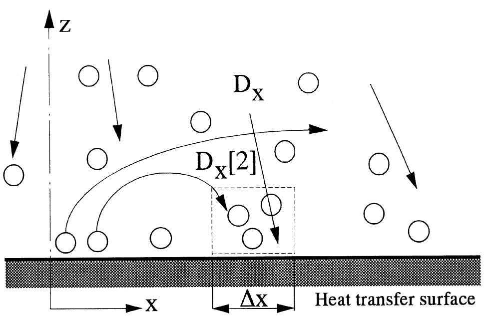

Fig. 1. Model of droplet behavior on hot surface.

2.2. Derivation of fundamental equations

From assumption (a), the local heat flux at x is given by

$$q_w[x] = q_{wl}[x] + q_{wa}[x] + q_{wr}[x]. \quad (1)$$

From condition (3), the last two terms of the right-hand side of this equation are given by

$$q_{wa}[x] = h_a[x](T_w - T_a), \quad q_{wr}[x] = q_{wr} = \varepsilon\sigma(T_w^4 - T_{rm}^4) \quad (2)$$

where h_a; heat transfer coefficient of induced air flow, T_a; temperature of air flow, ε; emmisivity of the plate, σ; the Stefan–Boltzmann constant, T_{rm}; room temperature.

Using assumption (b) and denoting the local heat flux at x upon impact of droplets after the $(j-1)$th rebound by $q_{dx}[j]$, $q_{wl}[x]$ is given by

$$q_{wl}[x] = q_{dx}[1] + q_{dx}[2] + q_{dx}[3] + \cdots. \quad (3)$$

Assumption (b) indicates that the diameter of droplets does not change during the rebound motion because evaporation upon impact is neglected. Then denoting the average diameter of droplets in the spray by d_m, $Q[j]$ is given by

$$Q[j] = C\left(\frac{\pi\rho c_p d_m^3 \Delta T_{sub}[j]}{6}\right) = C(1-C)^{j-1}\left(\frac{\pi\rho c_p d_m^3 \Delta T_{sub}[1]}{6}\right) \quad (4)$$

where ρ; density of the liquid, c_p; specific heat of the liquid, and $\Delta T_{sub}[j]$; the droplet subcooling before the jth impact.

2.3. Derivation of modeling equations of droplet heat flux for two-dimensional spray

Now, as shown in Fig. 1, consider a region of Δx in a two-dimensional spray. The center of the region is located at x. Denoting the number of droplets supplied for a unit time to the region after the $(j-1)$th rebound by $n_x[j]$, equation (3) is rewritten as

$$q_{wl}[x] = \left(\sum_{j=1} Q[j] n_x[j]\right)\left(\frac{1}{\Delta x}\right). \quad (5)$$

Substituting equation (4) to (5),

$$q_{wl}[x] = C\left\{\rho c_p \Delta T_{sub}[1] \sum_{j=1} (1-C)^{j-1} D_x[j]\right\} \quad (6)$$

where $D_x[j]$ is

$$D_x[j] = \left(\frac{\pi d_m^3}{6}\right)\left(\frac{n_x[j]}{\Delta x}\right). \quad (7)$$

Substituting equations (2) and (6) to (1),

$$q_w[x] = C\left\{\rho c_p \Delta T_{sub}[1] \sum_{j=1} (1-C)^{j-1} D_x[j]\right\} + h_a(T_w - T_a) + \varepsilon\sigma(T_w^4 - T_{rm}^4). \quad (8)$$

Taking $C = 0.5$ based on the report by Shoji et al. [10] and, equation (4) indicates

$$Q[3] = 0.25Q[1]. \quad (9)$$

So, for simplicity, neglecting the terms of $j > 2$ in equation (8), we obtain the following equation for the first term of the right hand side of equation (8).

$$q_{wl}[x] = C(\rho c_p \Delta T_{sub})\{D_x + (1-C)D_x[2]\} = q_{dx}[1] + q_{dx}[2] \quad (10)$$

where $\Delta T_{sub} = \Delta T_{sub}[1]$ and $D_x = D_x[1]$. Based on assumption (c), $D_x[2]$ in a two-dimensional spray is given by

$$D_x[2] = \frac{1}{L_{max}} \int_0^{L_{max}} D_x \, dx. \quad (11)$$

Substituting equation (11) to (10), finally the following equation is obtained for two-dimensional sprays.

$$q_{wl}[x] = C\rho c_p \Delta T_{sub} \left\{D_x + (1-C)\left(\frac{1}{L_{max}}\right)\int_0^{L_{max}} D_x \, dx\right\}. \quad (12)$$

This equation is valid only for $0 \leqslant x \leqslant L_{max}$ because $D_x[2]$ at $x > L_{max}$ is not given by equation (11).

2.4. Derivation of modeling equations of droplet heat flux for three-dimensional spray

Equations (8) and (10) hold also for three-dimensional sprays if x is replaced by r, but equation (11) should be changed. Taking the coordinate system as Fig. 2, $D_r[2]$ is given by

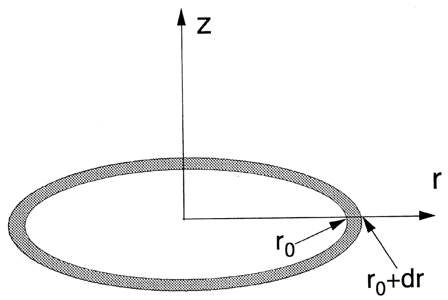

Fig. 2. Coordinate system for three dimensional spray.

$$D_r[2] = \left(\frac{1}{rL_{max}}\right)\int_0^r (r_0 D_{r0}) \, dr_0. \quad (13)$$

Substituting equation (13) to (10),

$$q_{wl}[r] = C\rho c_p \Delta T_{sub}\left\{D_r + (1-C)\left(\frac{1}{rL_{max}}\right)\int_0^r (r_0 D_{r0}) \, dr_0\right\}. \quad (14)$$

3. Determination of empirical parameters and conclusions

The total heat flux in the high temperature region of dilute sprays can be calculated by equations (1), (2) and (12) [or (14)] if the two empirical parameters, C and L_{max}, are determined. Fig. 3 shows the results of $q_{wl}[x]$, $q_{dx}[1]$, and $q_{dx}[2]$ predicted from equations (1), (2) and (12) for a two-dimensional spray of $a = 0.5$. Fig. 4 shows the same results for a three-dimensional spray. In these figures, the following normalized values are used. For the two-dimensional sprays,

$$q_{wl}[X]^* = \frac{q_{wl}[X]}{q_{wl}[X=0]} = \frac{q_{wl}[X]}{C\rho c_p \Delta T_{sub} D_{max}}$$

$$q_{dX}[1]^* = \frac{q_{dX}[1]}{q_{wl}[X=0]} = \frac{C\rho c_p \Delta T_{sub} D_X}{C\rho c_p \Delta T_{sub} D_{max}} = \frac{D_X}{D_{max}}$$

$$q_{dX}[2]^* = \frac{q_{dX}[2]}{q_{wl}[X=0]} = \frac{(1-C)\left(\frac{1}{L_{max}^*}\right)\int_0^X D_X \, dX}{D_{max}}$$

$$q_{wl}[X]^* = q_{dX}[1]^* + q_{dX}[2]^* \quad (15)$$

and $X = x/b$, $L_{max}^* = L_{max}/b$, and $D_{max} = D_{x=0}$. b is the half width of the distribution of droplet flow rate, D_x, for two-dimensional sprays defined by

$$\frac{D_x}{D_{max}} = \exp\left[-a\left(\frac{x}{b}\right)^2\right]. \quad (16)$$

For three-dimensional sprays,

$$q_{wl}[R]^* = q_{dR}[1]^* + q_{dR}[2]^*. \quad (17)$$

Here, $R = r/b$ and b for three-dimensional sprays is defined by

$$\frac{D_r}{D_{max}} = \exp\left[-a\left(\frac{r}{b}\right)^2\right]. \quad (18)$$

Comparing Fig. 3 with Fig. 4, it is found that the contribution of rebounded droplets ($q_{wX}[2]^*$ and $q_{dR}[2]^*$) to the droplet heat flux ($q_{wl}[X]^*$ and $q_{wl}[R]^*$) is more remarkable in two-dimensional sprays than three-dimensional sprays.

Based on the result mentioned above, the empirical parameters, C and L_{max}, were determined independently by using the experimental data of $q_w[x]$ of two-dimen-

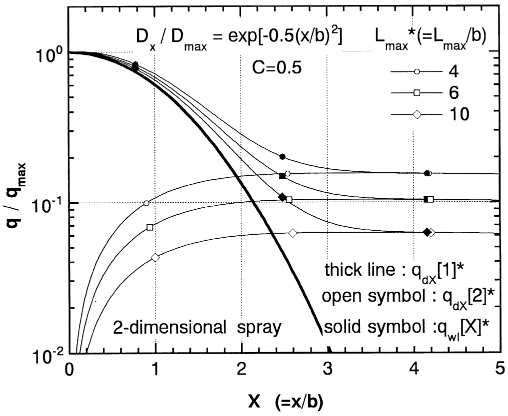

Fig. 3. Predicted results of heat flux distribution for two-dimensional sprays ($a = 0.5$).

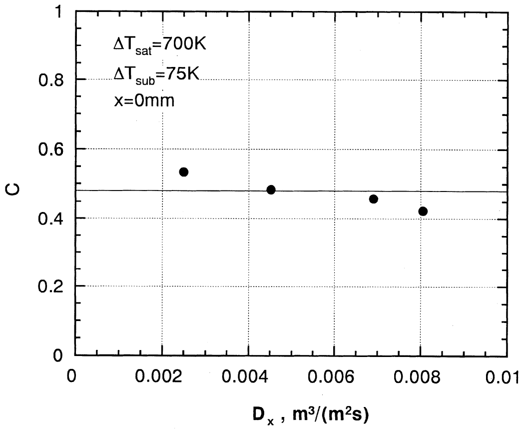

Fig. 5. Estimated values of C at stagnation point of spray ($\Delta T_{sat} = 700$ K).

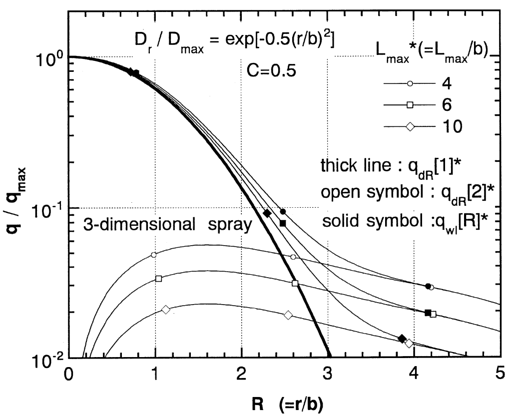

Fig. 4. Predicted results of heat flux distribution for three-dimensional sprays ($a = 0.5$).

sional sprays obtained in the previous study [1]. Here, at $x = 0$, equation (12) becomes

$$q_{wl}[x=0] = C(\rho c_p \Delta T_{sub}) D_{x=0}. \quad (19)$$

Since, in the previous study, the spray flow was formed without using pressurized air, the contribution of air flow to the total heat flux is considered small. So, taking $h_a = 0$ and using an appropriate value of ε in equation (2), $q_{wl}[x = 0]$ in equation (19) can be calculated from experimental data of $q_w[x = 0]$ and then the value of C can be determined from equation (19). The value of C obtained by using the experimental data of $q_w[x = 0]$ at $\Delta T_{sat} = 700$ K are plotted in Fig. 5 for four droplet flow rates, $D_{x=0}$. It is found from the figure that the value of C is almost independent of the droplet flow rate and the averaged value is $C = 0.48$. This value is almost the same with the value reported for single droplets by Shoji et al. [10] ($C = 0.5$). These results indicate that assumption (b) is valid at least as the first order approximation.

Next, modifying equation (12), we obtain

$$L_{max} = \frac{(1-C)\int_0^x D_x \mathrm{d}x}{\left\{\dfrac{q_{wl}[x]}{C(\rho c_p \Delta T_{sub})}\right\} - D_x}. \quad (20)$$

Using $C = 0.48$ and the data of $q_{wl}[x]$ in the stagnation region ($0 < x < 30$ mm) at $\Delta T_{sat} = 700$ K, the value of L_{max} was calculated from equation (20). The results obtained are plotted in Fig. 6. It is found from the figure that, although the values of L_{max} scatter a little, they are almost independent of x and the total droplet flow rate (Q_d) and the averaged value is about $L_{max} = 0.06$ m. This value of L_{max} is not so different from the experimental

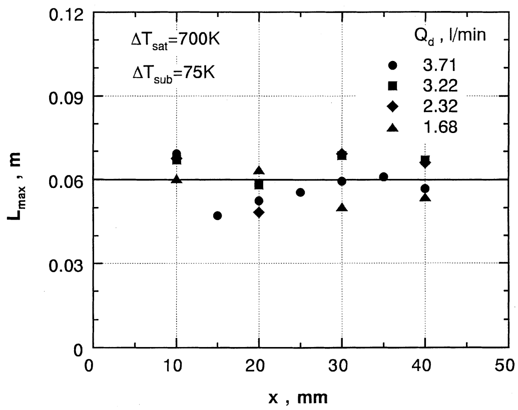

Fig. 6. Estimated values of L_{max} at stagnation point of spray ($\Delta T_{sat} = 700$ K).

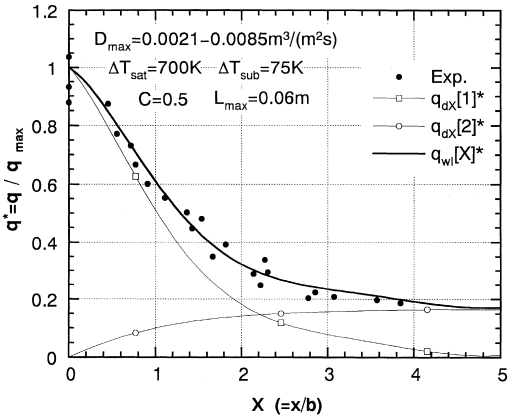

Fig. 7. Comparison between experimental data and model predictions for dependence of local heat flux of two-dimensional spray on distance from stagnation point ($\Delta T_{sat} = 700$ K).

value of the representative length of the stagnation region obtained for the two-dimensional spray used in the previous report [1]. These results indicate that assumption (c) is valid also at least as the first order approximation.

The normalized values, $q_{wl}[X]^*$, $q_{dX}[1]^*$ and $q_{dX}[2]^*$, predicted from the present model with the values of C and L_{max} are plotted to the normalized distance X together with experimental data [1] in Fig. 7. In Fig. 8, the values of $q_w[x]$ predicted from the present model are plotted to the surface superheat. It is found from these figures that the experimental data of $q_{wl}[X]^*$ is much

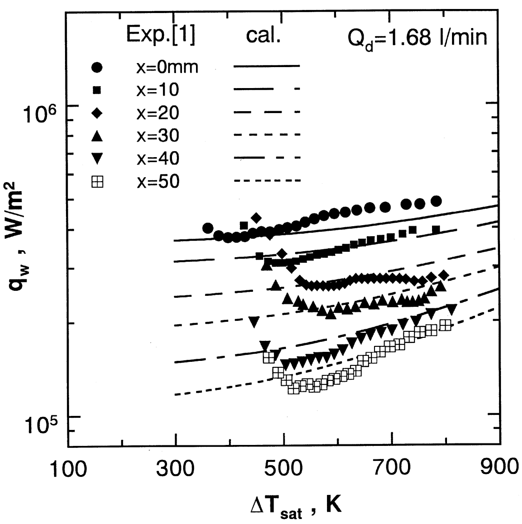

Fig. 8. Comparison between experimental data and model predictions for dependence of local heat flux of two-dimensional spray on surface superheat.

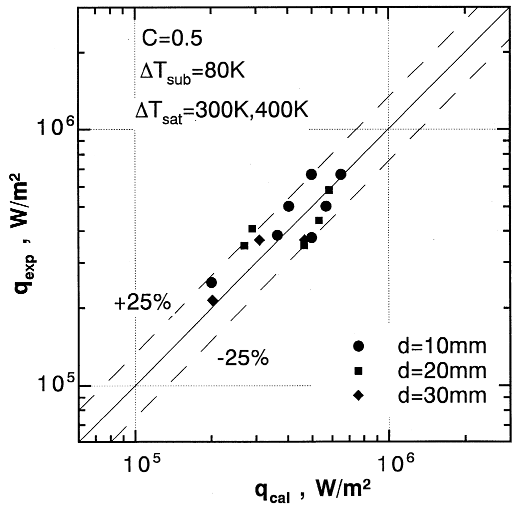

Fig. 9. Comparison between experimental data and model predictions for three-dimensional spray.

larger than $q_{dX}[1]^*$ and the predicted dependence of local heat flux on surface superheat is in reasonable agreement with the experimental data.

As far as the present authors know, there is no experimental result for the heat flux distribution of three-dimensional sprays. For mist flows of water atomized by air, however, Ohkubo and Nishio [11] reported experimental data for the effects of the surface diameter on the averaged heat transfer coefficient. In their experiments, the diameter was changed from $d = 10$–30 mm and they reported the following distribution of mass flow velocity of droplets.

$$\frac{D_r}{D_{max}} = \exp\left[-0.693\left(\frac{r}{b}\right)^2\right]. \tag{21}$$

In their report, the value of b is given as an empirical equation. Substituting equation (21) into (14), the distribution of $q_{wl}[r]$ on a circular plate can be calculated. Using this distribution and equation (1), the distribution of $q_w[r]$ can be calculated. Averaging this distribution of $q_w[r]$ over the plate of diameter d, the averaged heat flux is calculated. The experimental results, q_{exp}, are plotted to the predicted results, q_{cal}, in Fig. 9. It is found from the figure that the deviation of the experimental data from the calculated results is within 25% and the present model works well also for three-dimensional sprays.

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